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The role of nucleon recoil in low-energy antikaon-deuteron scattering

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Abstract

The effect of the nucleon recoil for antikaon-deuteron scattering is investigated in the framework of effective field theory. In particular, we concentrate on the calculation of the nucleon recoil effect for the double-scattering process. It is shown that the leading correction to the static term that emerges at order $\xi^{1/2}$ with $\xi = M_K/m_N$ vanishes due to a complete cancellation of individually large contributions. The resulting recoil effect in this process is found to be of order of 10-15% as compared to the static term. We also briefly discuss the application of the method in the calculations of the multiple-scattering diagrams.

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1 Introduction

Recent combined analyses of pionic hydrogen and pionic deuterium have revealed a significant progress towards a precise extraction of the pion-nucleon interaction parameters at threshold, see Ref. [1] and also [2, 3] for review articles. One might expect that the properties of the $\bar{K}N$ interaction can be investigated in a similar way using a combined analysis of kaonic hydrogen and kaonic deuterium. The data on this sort of bound systems are provided by the ongoing experiment of the DEAR/SIDDHARTA collaboration, which aims to measure the $1s$ energy level shift and width of kaonic hydrogen and kaonic deuterium, eventually with an accuracy of several eV [4]. Contrary to the πN case, the $\bar{K}N$ scattering length, however, is known to be large (of order of 1 fm) and strongly absorptive. This means that in this case one will need to measure 4 independent quantities (the real and imaginary parts of the S-wave $\bar{K}N$ scattering lengths b_0, b_1), whereas for πN scattering one has to deal with two real scattering lengths. Consequently, the role of the deuterium measurements is rather different in these two cases. While the pion-nucleon scattering lengths can, at least in principle, be determined from the pionic hydrogen alone with the data from pionic deuterium serving as an additional consistency check, both data from the kaonic hydrogen and kaonic deuterium are needed to determine the values of b_0, b_1.

The analysis of the experimental data proceeds as follows. From the measured values of the energy shift and width of the ground state in the kaonic hydrogen (kaonic deuterium) one first extracts the threshold scattering amplitudes of K^-p (K^-d) scattering by using the DGBT-type formulae [5] at next-to-leading order in isospin breaking (see, e.g., [2, 6, 7])

$$
\begin{aligned}
\Delta E_{1s} - i\,\frac{\Gamma_{1s}}{2} &= -2\alpha^3\mu^2 a_p(1-2\mu\alpha(\ln\alpha-1)a_p)\,, \\
\Delta E_{1s}^d - i\,\frac{\Gamma_{1s}^d}{2} &= -2\alpha^3\mu_d^2 A_{\bar{K}d}\biggl(1-2\mu_d\alpha(\ln\alpha-1)A_{\bar{K}d}\biggr),
\end{aligned}
\tag{1}
$$

where μ (μ_d) stands for the reduced mass of the $\bar{K}N$ ($\bar{K}d$) system and a_p ($A_{\bar{K}d}$) refers to the pertinent threshold amplitudes[1]. Here and in what follows, we adopt the notation of Ref. [2].

In the next step, one has to express the quantities a_p and $A_{\bar{K}d}$ in terms of the S-wave $\bar{K}N$ scattering lengths b_0, b_1 which are defined in the isospin-symmetric limit at $\alpha = 0$ and $m_d - m_u = 0$. The isospin structure of the $\bar{K}N$ scattering amplitude in this world is proportional to $b_0 + b_1\vec{\tau}_K\cdot\vec{\tau}_N$ and the relation to the scattering lengths a_0, a_1 corresponding to the total isospin $I = 0, 1$ is given by $a_0 = b_0 - 3b_1$, $a_1 = b_0 + b_1$. Our ultimate goal is to extract the precise values of b_0, b_1 from the experiment which are then to be confronted with the theoretical predictions obtained in the unitarized ChPT [9]. This would provide a beautiful test of our knowledge about the $SU(3)$ meson-baryon dynamics at low energy. Eventually, it will be very interesting to compare the experimental results with the scattering lengths directly extracted from lattice QCD with the use of the recently proposed method [10].

In the isospin-symmetric world, $a_p = b_0 - b_1$. This relation is strongly modified when isospin is broken due to the unitary cusp effect [6, 11]. However, the corrections due to the cusp effect can be resummed to all orders in perturbation theory leading to the modified expression for a_p which is still given in terms of b_0, b_1 and the physical masses of the kaons and nucleons [6, 11]. Thus, the presence of the cusp effect in a_p does not affect the accuracy of the extraction of b_0, b_1 from the data.

[1]In the case of kaonic atoms, higher-order Coulomb corrections may turn out to be not completely negligible numerically as shown, e.g., in Ref. [8] through the exact solution of the Schrödinger equation for the bound state. This issue is, however, relatively easy to cure since the large contribution comes from an iteration of a particular diagram to all orders. Replacing the factor $1-2\mu\alpha(\ln\alpha-1)a_p$ by $(1+2\mu\alpha(\ln\alpha-1)a_p)^{-1}$ already captures the bulk of the effect. We shall not further elaborate on this issue.

The situation is rather different for the kaonic deuterium, even in the absence of isospin breaking. In order to perform the analysis of the data, the quantity $A_{\bar{K}d}$ should be expressed in terms of b_0, b_1 in some kind of multiple-scattering expansion. The procedure is plagued by our poor knowledge of the 3-body $\bar{K}d$ interactions. It is not clear a priori whether the corresponding uncertainties are large enough to preclude one from being able to extract the values of b_0, b_1 from experimental data if such precise data would be available. A systematic model-independent analysis of the uncertainties in the 3-body sector is therefore needed in order to properly analyze the kaonic deuterium data, which will be provided by SIDDHARTA collaboration in the near future.

The aim of the present work is to address the recoil effect, which is one of the major sources of the theoretical uncertainty in the study of $\bar{K}d$ scattering at low energy. Our paper is organized as follows. In section 2 we briefly review the theoretical framework, which will be used to systematically investigate recoil effect in the $\bar{K}d$ scattering. The background information about the recoil effect is given in section 3. Section 4 is devoted to the discussion of the recoil effect in the double-scattering diagram and contains both the analytic calculations and numerical results. Recoil effect in the multiple-scattering diagrams is briefly discussed in section 5. Finally, our conclusions are presented in section 6.

2 The framework

During the last few decades, the $\bar{K}d$ scattering at low energy has been addressed on numerous occasions within the framework of Faddeev equations, see e.g. Refs.[12, 13, 14, 15] for some of those works. It should, however, be pointed out that the results of these beautiful calculations are of no direct use in the analysis of the SIDDHARTA data since these calculations do not provide an explicit relation between $A_{\bar{K}d}$ and b_0, b_1, which is needed for the analysis.

Contrary to the brute-force Faddeev calculations, the multiple-scattering series for the $\bar{K}d$ scattering length, see e. g. [7, 16, 17], can be utilized for the analysis of the data. However, as it is well known, the $\bar{K}N$ scattering lengths are large due to the presence of the subthreshold $\Lambda(1405)$ resonance, and the multiple-scattering series does not converge. The resummation of the series can be carried out analytically using the so-called fixed center approximation (FCA) in which nucleons are considered as static sources, i.e. $m_N \to \infty$. However, since $M_K/m_N \simeq 0.5$, one may *a priori* expect large corrections to the static limit. To the best of our knowledge, no systematic method to evaluate these *recoil corrections* exists in the case of a large scattering length, where the multiple-scattering series should be resummed. The present paper aims at the formulation of the framework, which is capable to address this problem.

A comparison of the exact numerical solutions of the Faddeev equations for potential models with the multiple-scattering series resummed in the FCA reveals a pretty good agreement in most cases, see [18] and references therein. This observation provides motivation for our approach and serves as a starting point since it indicates that the recoil corrections could be rather small even at $M_K/m_N \simeq 0.5$ and might be amenable to the perturbative treatment. Note that the perturbative treatment of the recoil corrections becomes indispensable if the resummed multiple-scattering series is considered. The potential model is a useful testing ground for our framework, since it allows to examine the convergence of the perturbative series towards the exactly known result.

As pointed out in Ref. [7], the non-relativistic effective field theory (EFT) provides an ideal tool to explore the multiple-scattering expansion. In particular, one readily reproduces the known result [16, 17] for static nucleons, and the inclusion of the recoil effect is straightforward (at least, formally). Prior to going to the details of the calculation we find it appropriate to discuss the essential features of the EFT approach to the problem of interest.

i) The usefulness of the multiple-scattering expansion for the $\bar{K}d$ scattering is due to the ap-

pearance of two distinct momentum scales. Whereas the NN interactions and 3-body $\bar{K}NN$ interactions are mediated at large distances by 1-pion exchange, the dominant long-distance contribution to the $\bar{K}N$ scattering is governed by 2-pion exchange. For this reason, we may treat $\bar{K}N$ interactions as point-like, whereas NN and $\bar{K}NN$ interactions are described by non-local potentials.

ii) The fact that $\Lambda(1405)$ resonance is located close to the $\bar{K}N$ threshold can, potentially, lead to enhancement of the non-local contributions in the $\bar{K}N$ scattering amplitudes. One may expect, however, that the non-local effects could be still taken into account perturbatively, by using the effective-range expansion of the $\bar{K}N$ amplitude. This assumption can be checked *a posteriori* by explicit calculations.

iii) The expansion parameter associated with the $\bar{K}N$ interaction is given by $b \cdot \langle 1/r \rangle \simeq 1$, where b denotes the S-wave $\bar{K}N$ scattering length and $\langle 1/r \rangle \simeq 0.5\ \mathrm{fm}^{-1}$ is the expectation value of the operator r^{-1} between the deuteron wave functions. Since the expansion parameter is large, the multiple-scattering series does not converge and should be resummed to all orders.

iv) At the energies which are relevant for the problem in question, effects due to explicit creation/annihilation of the particles can be neglected. These processes are taken into account implicitly through various low-energy constants and non-local interactions present in the Lagrangian. The non-relativistic EFT, which we use, conserves the hadron number explicitly.

v) In addition, we expect that there is no need to include hyperonic channels $\bar{K}NN - \pi YN$ with $Y = \Lambda, \Sigma$ explicitly in the non-relativistic EFT framework. This conjecture is backed by the numerical coupled-channel Faddeev calculations [12, 13, 15, 18].

vi) The (non-local) NN potential can be directly imported from chiral effective field theories, see, e.g., [19, 20, 21] for recent review articles. The NN and $\bar{K}N$ sectors of the non-relativistic EFT do not talk to each other. However, in this work for demonstrative purposes we shall use NN potential in the separable form.

vii) Inclusion of the three-body force is required in EFT for the consistency reasons alone. In order to estimate the numerical strength of the three-body force notice that the total two-nucleon absorption rate in K^-d scattering amounts for $(1.22 \pm 0.09)\%$ [22]. Assuming that the dispersive and absorptive parts of the 3-body force have the same order of magnitude, we may expect effects of the 3-body contribution to $A_{\bar{K}d}$ at a few percent level. This is definitely beyond the theoretical accuracy of the present calculation. For comparison, note that the imaginary part of the πd scattering length, which is dominated by the contribution from $\pi d \to NN$ breakup reaction, amounts more than 20% of the real part [23].

viii) As we shall see below, the use of the EFT framework enables one to systematically obtain the expansion of $A_{\bar{K}d}$ in powers of the inverse nucleon mass. The leading-order term corresponds to the static nucleon limit with $m_N \to \infty$. The key assumption is that this expansion is perturbative, even if the multiple-scattering series is not.

ix) In order to simplify the bookkeeping, we neglect relativistic effects for the time being. Stated differently, we consider EFT of the underlying fundamental theory which is assumed to be the non-relativistic potential model. All general conclusions should remain in place for such a model. A systematic inclusion of the relativistic corrections will be discussed elsewhere. In the non-relativistic case at hand one may introduce the dimensionless parameter $\xi = M_K/m_N$ and consider the expansion of the amplitude $A_{\bar{K}d}$ in this parameter. Both integer and half-integer powers of ξ appear in this expansion, emerging from different momentum scales.

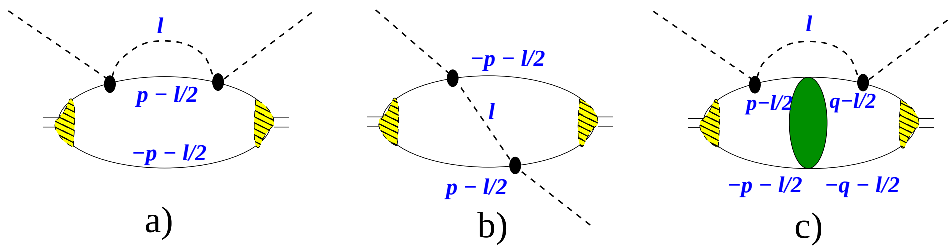

Figure 1: Diagrams corresponding to the meson double scattering on the nucleons in the deuteron

x) For the time being, we completely neglect isospin-breaking effects in $\bar{K}d$ scattering. They will be considered in a later publication.

Equipped with this general clue, we now consider the perturbative calculation of the recoil corrections in detail.

3 Some known facts about nucleon recoil

Historically, the appearance of sizable cancellations in the case of the πd scattering length was first pointed out in the papers by Kolybasov et al. [24] and, independently, by Fäldt [25] who argued that the naive static term provides a good approximation of rescattering effects. Recently, the role of the recoil effects was investigated for πd scattering and for the reaction $\gamma d \to \pi^+ nn$ within EFT [26, 27]. In particular, it was shown that the 3-body singularity that occurs in the πNN intermediate state is the origin of non-analytic corrections to the static term of order $\sqrt{M_\pi/m_N}$. It was also found that the importance of the recoil effect is directly connected with Pauli selection rules for the intermediate NN state. In the case when the S-wave NN interaction is forbidden by the Pauli principle, the leading $\sqrt{M_\pi/m_N}$ correction cancels in the diagrams of double πN scattering, diagrams a) and b) in Fig. 1, thus resulting in the small net contribution. The situation is different in case when the S-wave NN interaction is allowed, see diagram c) in Fig. 1. First, the leading $\sqrt{M_\pi/m_N}$ correction does not vanish here (the contributions from diagrams a) and b) appear with the same sign). For this reason, it was concluded in [26, 27] that recoil effects should be significant in this case. Secondly, the additional diagram with the S-wave NN interaction in the intermediate state must also be taken into account in the calculation. In the process $\gamma d \to \pi^+ nn$, the intermediate NN interaction appears to be in the ${}^1S_0$ partial wave. Since NN interaction in the ${}^1S_0$ partial wave differs considerably to the one in the ${}^3S_1$ channel there is no *a priori* reason to expect any kind of cancellation between the recoil correction from diagrams similar to a) and b) and the contribution of diagram c). However, for πd and Kd scattering the intermediate and final NN interaction occurs in the same channel (${}^3S_1 - {}^3D_1$). Therefore, a combined consideration of all diagrams of Fig.1 is needed to draw a definite conclusion about the recoil effect[2]. Note that the Pauli-allowed recoil correction for πd scattering gets multiplied with the isoscalar πN scattering length squared which renders this effect negligible. This is, however, not the case for $\bar{K}d$ scattering where both the isoscalar and isovector interactions are of a similar (and large) size.

[2]We neglect the D-wave contribution in the following. This does not affect the structure of the recoil contribution, which is of our primary interest here.

4 $\bar{K}d$ scattering: Recoil effect in the double-scattering process

4.1 The method

Despite the well-known fact that the multiple-scattering series for the $\bar{K}d$ scattering requires resummation to all orders, at the first stage of our investigation we concentrate on the double-scattering diagram. The aim is to demonstrate the technique used to obtain a systematic expansion of any Feynman diagram in (generally non-integer) powers of the parameter ξ. To this end, we apply the perturbative uniform expansion method proposed in Ref. [28]. The same results can be also obtained using the threshold expansion method for the Feynman diagrams developed in Ref. [29]. The advantage of the scheme of Ref. [28] is, however, that it is not tied to a particular (dimensional) regularization.

Application of this method to carry out the integral in an arbitrary one-loop Feynman diagram[3] can be summarized as follows:

i) Identify the relevant momentum scales and separate the range of integration into the regions according to these scales. Suppose, for instance, that there are two distinct scales: the small scale η and the large scale Λ, $\eta/\Lambda \ll 1$. Then, the integrand $f(\eta, q, \Lambda)$ with q referring to the integration momentum has three relevant regimes:

   1. The low-momentum regime with $\eta \sim q \ll \Lambda$,
   2. The high-momentum regime with $\eta \ll q \sim \Lambda$,
   3. The intermediate regime with $\eta \ll q \ll \Lambda$.

ii) In each region, perform the Taylor expansion of the integrand $f(\eta, q, \Lambda)$ in the corresponding small parameters.

iii) Then, the original integrand fulfills the equality

$$f(\eta, q, \Lambda) = f_{\mathrm{l}}(\eta, q, \Lambda) + f_{\mathrm{h}}(\eta, q, \Lambda) - f_{\mathrm{i}}(\eta, q, \Lambda) \tag{2}$$

at the given order in η/Λ. Here, $f_{\mathrm{l}}, f_{\mathrm{h}}, f_{\mathrm{i}}$, refer to the Taylor-expanded integrand f in the low-, high- and intermediate-momentum regimes, respectively. In particular, the integrand f_{i}, which represents the intermediate region, contains infrared- and ultraviolet-divergent terms necessary to make the integrals over the functions f_{l} and f_{i} finite and thus plays the role of a regulator.

iv) Finally, integrate the functions $f_{\mathrm{l}}, f_{\mathrm{h}}, f_{\mathrm{i}}$ over the whole momentum range. Since the above combination of these functions reproduces the original function $f(\eta, q, \Lambda)$ at the given order in η/Λ, the same combination of integrals will yield the correct result for the original integral up to the same order.

Let us now apply this method to the double $\bar{K}N$ scattering diagrams of Fig. 1. The corresponding contribution to the $\bar{K}d$ scattering length reads, see Ref. [26] for more details:

$$A_{\bar{K}d}^{\text{doubl. scatt.}} = \frac{8\pi\mu_d M_K}{\mu^2}\left(R_a + R_b + R_c\right), \tag{3}$$

[3] The method is, however, not restricted to one-loop diagrams.

where the quantities R_a, R_b and R_c are defined as follows:

$$
\begin{aligned}
R_a &= \frac{b_0^2+3b_1^2}{2M_K}\int\frac{d^3\mathbf{p}\,d^3\mathbf{l}}{(2\pi)^6}\,\Psi^2\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\left(\left[\frac{\mathrm{l}^2}{2M_K}+\frac{\mathrm{p}^2+\gamma^2}{m_N}+\frac{\mathrm{l}^2}{4m_N}\right]^{-1}-\left[\frac{\mathrm{l}^2}{2\mu}\right]^{-1}\right),\\
R_b &= \frac{b_0^2-3b_1^2}{2M_K}\int\frac{d^3\mathbf{p}\,d^3\mathbf{l}}{(2\pi)^6}\,\Psi\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\Psi\left(\mathbf{p}-\frac{\mathbf{l}}{2}\right)\left[\frac{\mathrm{l}^2}{2M_K}+\frac{\mathrm{p}^2+\gamma^2}{m_N}+\frac{\mathrm{l}^2}{4m_N}\right]^{-1},\\
R_c &= \frac{b_0^2}{4m_N^2M_K}\int\frac{d^3\mathbf{p}\,d^3\mathbf{q}\,d^3\mathbf{l}}{(2\pi)^9}\,\Psi\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\Psi\left(\mathbf{q}+\frac{\mathbf{l}}{2}\right)M_{NN}(\mathbf{p},\mathbf{q},E(\mathrm{l}))\\
&\times\left[\frac{\mathrm{l}^2}{2M_K}+\frac{\mathrm{p}^2+\gamma^2}{m_N}+\frac{\mathrm{l}^2}{4m_N}\right]^{-1}\left[\frac{\mathrm{l}^2}{2M_K}+\frac{\mathrm{q}^2+\gamma^2}{m_N}+\frac{\mathrm{l}^2}{4m_N}\right]^{-1}
\end{aligned}
\tag{4}
$$

where Ψ denotes the deuteron wave function, $\gamma^2=m_N\varepsilon_d$ with ε_d being the deuteron binding energy and the energy $E(\mathrm{l})$ is defined as

$$
E(\mathrm{l})=-\varepsilon_d-\frac{\mathrm{l}^2}{2M_K}-\frac{\mathrm{l}^2}{4m_N}\,. \tag{5}
$$

Further, M_{NN} denotes the NN amplitude. Its normalization is chosen such that in the CM frame, the amplitude M_{NN} evaluated on the energy shell is related to the S-wave scattering phase shift $\delta(k)$ through $M_{NN}(k,k,k^2/m_N)=16\pi m_N(k\cot\delta(k)-ik)^{-1}$. In the quantity R_a, renormalization of the one-loop $\bar{K}N$ scattering amplitude is carried out by performing the subtraction at threshold. At this order, this prescription yields the same result as dimensional regularization, but has the advantage that it is not tied to a particular regularization scheme.

In the static limit with $\xi=M_K/m_N\to 0$, only the amplitude R_b survives. In the vicinity of the static limit, each of the amplitudes R_i, $i=a,b,c$ can be expanded in half-integer powers of ξ

$$
R_i=R_i^{\mathsf{stat}}+\xi^{1/2}\,R_i^{(1)}+\xi\,R_i^{(2)}+\xi^{3/2}\,R_i^{(3)}+\ldots\,. \tag{6}
$$

Our aim is to perform this expansion systematically using the perturbative uniform expansion method [28]. In order to do this, it is convenient to rewrite the amplitudes R_i by showing the explicit dependence on the parameter ξ in the integrands

$$
R_a+R_b+R_c=b_0^2\,(I_{\mathsf{st}}+I_0+I_{NN}+\Delta I_{\mathsf{st}})-3b_1^2\,(I_{\mathsf{st}}-I_1+\Delta I_{\mathsf{st}})\,, \tag{7}
$$

where

$$
I_{0(1)}=\int\frac{d^3\mathbf{p}\,d^3\mathbf{l}}{(2\pi)^6}\left[\Psi^2\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\pm\Psi\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\Psi\left(\mathbf{p}-\frac{\mathbf{l}}{2}\right)\right]\left(\frac{1}{\mathrm{l}^2+\xi\left(2(\mathrm{p}^2+\gamma^2)+\mathrm{l}^2/2\right)}-\frac{1}{\mathrm{l}^2(1+\xi)}\right), \tag{8}
$$

$$
I_{NN}=\frac{\xi}{m_N}\int\frac{d^3\mathbf{p}\,d^3\mathbf{q}\,d^3\mathbf{l}}{(2\pi)^9}\frac{\Psi\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\Psi\left(\mathbf{q}+\frac{\mathbf{l}}{2}\right)M_{NN}(\mathbf{p},\mathbf{q},E(\mathrm{l}))}{\left[\mathrm{l}^2+\xi\left(2(\mathrm{p}^2+\gamma^2)+\mathrm{l}^2/2\right)\right]\left[\mathrm{l}^2+\xi\left(2(\mathrm{q}^2+\gamma^2)+\mathrm{l}^2/2\right)\right]}, \tag{9}
$$

$$
I_{\mathsf{st}}=\int\frac{d^3\mathbf{p}\,d^3\mathbf{l}}{(2\pi)^6}\,\Psi\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\Psi\left(\mathbf{p}-\frac{\mathbf{l}}{2}\right)\frac{1}{\mathrm{l}^2}\,,\qquad \Delta I_{\mathsf{st}}=\frac{-\xi}{1+\xi}I_{\mathsf{st}}\,. \tag{10}
$$

Here, I_{st} corresponds to the FCA result (obtained in the static limit). The recoil corrections corresponding to the Pauli-allowed (forbidden) S-wave NN intermediate state in the diagrams a) and b) of

Fig. 1 are given by the integrals I_0 and $\Delta I_{\sf st}$ (I_1 and $\Delta I_{\sf st}$) in Eqs. (8) and (10). For the Pauli-allowed NN state there is also a contribution from the diagram c) given by the integral I_{NN}.

We define the recoil corrections as $\Delta I_1 = -I_1 + \Delta I_{\sf st}$ and $\Delta I_0 = I_0 + I_{NN} + \Delta I_{\sf st}$ for isovector and isoscalar $\bar{K}N$ interactions, respectively. Below, we demonstrate how these corrections can be evaluated by using the method of Ref. [28].

Consider, for instance, the integral I_1, which can be rewritten in the following form

$$I_1 = \frac{\xi}{1+\xi}\int \frac{d^3\mathbf{p}d^3\mathbf{l}}{(2\pi)^6}\frac{f(\mathbf{p},\mathbf{l})}{\mathrm{l}^2}, \qquad f(\mathbf{p},\mathbf{l}) = \Phi(\mathbf{p},\mathbf{l})\frac{\mathrm{l}^2/2 - b^2}{\mathrm{l}^2 + \xi b^2 + \xi \mathrm{l}^2/2}\,. \tag{11}$$

Here, $b^2 = 2(\mathrm{p}^2 + \gamma^2)$ and the quantity Φ denotes the following combination of the wave functions

$$\Phi(\mathbf{p},\mathbf{l}) = \Psi^2\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right) - \Psi\left(\mathbf{p}+\frac{\mathbf{l}}{2}\right)\Psi\left(\mathbf{p}-\frac{\mathbf{l}}{2}\right)\,. \tag{12}$$

There are three relevant momentum regimes in the integral given by Eq. (11):

*1. The low-*l *regime.*
In this case, the involved momenta scale as follows:

$$\frac{\mathrm{l}^2}{2M_K} \sim \frac{\mathrm{p}^2}{2m_N} \quad\Longrightarrow\quad \mathrm{l} \sim \sqrt{\xi}\,\mathrm{p}, \quad \mathrm{p} \sim \langle 1/r\rangle_{\sf wf}\,, \tag{13}$$

where $\langle\ldots\rangle_{\sf wf}$ denotes averaging over the deuteron wave functions. In this regime, $\sqrt{\xi}b \sim \mathrm{l}$ and $\mathrm{l} \ll b \sim \mathrm{p}$. Consequently, there are two different expansion parameters: ξ and l^2/b^2. Note, however, that the term ξb^2 occurring in the propagator of Eq. (11) is of the same order as l^2 and thus should be kept unexpanded. Expanding the wave functions in Eq. (12) in powers of the momentum $\mathbf{l}$ and using $\Phi(\mathbf{p},\mathbf{0}) = 0$ we get

$$\Phi(\mathbf{p},\mathbf{l}) = \frac{1}{2}\,\mathrm{l}_i\mathrm{l}_j\nabla_{\mathrm{l}}^i\nabla_{\mathrm{l}}^j\Phi(\mathbf{p},\mathbf{l})\Big|_{\mathbf{l}=\mathbf{0}} + \ldots\,. \tag{14}$$

Note that we only keep terms even in l in the expansion of $\Phi(\mathbf{p},\mathbf{l})$ since all odd terms will vanish after performing angular integration in Eq. (11). Expanding the integrand $f(\mathbf{p},\mathbf{l})$ and averaging over the directions according to $\mathrm{l}_i\mathrm{l}_j \to \delta_{ij}\mathrm{l}^2/3$ leads to

$$f_{\mathrm{l}}(\mathbf{p},\mathbf{l}) = \Phi_2(\mathbf{p})\left(-b^2 + \mathrm{l}^2/2 + \frac{\xi b^4}{\mathrm{l}^2+\xi b^2}\right) + \ldots \qquad \text{with} \qquad \Phi_2(\mathbf{p}) \equiv \frac{1}{6}\nabla_{\mathrm{l}}^2\Phi(\mathbf{p},\mathbf{l})\Big|_{\mathrm{l}=0}\,, \tag{15}$$

where the ellipses refer to terms of a higher order in ξ. We attach the subscript "l" to the expanded integrand in order to signify the low-l regime.

*2. The high-*l *regime.*
In this case, the momenta l and p scale as follows:

$$\mathrm{l} \sim \mathrm{p} \sim \langle 1/r\rangle_{\sf wf} \quad\Longrightarrow\quad \sqrt{\xi}b \ll \mathrm{l} \sim b\,. \tag{16}$$

This implies that the function $f(\mathbf{p},\mathbf{l})$ can be safely expanded in powers of ξ:

$$f_{\mathrm{h}}(\mathbf{p},\mathbf{l}) = \Phi(\mathbf{p},\mathbf{l})\left(\frac{-b^2+\mathrm{l}^2/2}{\mathrm{l}^2} + \xi\frac{b^4 - \mathrm{l}^4/4}{\mathrm{l}^4}\right) + \ldots\,. \tag{17}$$

We attach the subscript "h" to the expanded integrand in the high-l regime.

3. The intermediate regime.
This case corresponds to the scaling $\sqrt{\xi} b \ll \mathrm{l} \ll b$. The expanded integrand f_{i} in this regime can be obtained by using the heavy-baryon expansion to the function f_{l} or the low-momentum expansion to the function f_{h}. Both expansions lead, of course, to the same result

$$f_{\mathrm{i}}(\mathbf{p},\mathbf{l}) = \Phi_2(\mathbf{p}) \left(-b^2 + \mathrm{l}^2/2 + \xi \frac{b^4}{\mathrm{l}^2} \right) + \dots \,. \tag{18}$$

Adding up the contributions from all three different regimes we obtain

$$f_{\mathrm{l}} + f_{\mathrm{h}} - f_{\mathrm{i}} = \Phi(\mathbf{p},\mathbf{l}) \frac{\mathrm{l}^2/2 - b^2}{\mathrm{l}^2} + \xi \left(\Phi(\mathbf{p},\mathbf{l}) \frac{b^4 - \mathrm{l}^4/4}{\mathrm{l}^4} - \Phi_2(\mathbf{p}) \frac{b^4}{\mathrm{l}^2} \right) + \xi \, \frac{\Phi_2(\mathbf{p}) b^4}{\mathrm{l}^2 + \xi\, b^2} + \dots \,. \tag{19}$$

It is now easy to identify the powers of ξ which emerge after carrying out the integration. The first two terms are polynomials in ξ. Recalling that the whole integral is multiplied by a factor $\xi/(1+\xi)$, it is seen that these terms start to contribute at $\mathcal{O}(\xi)$ and $\mathcal{O}(\xi^2)$, respectively. Rescaling the integration momentum $\mathrm{l} \to \sqrt{\xi}\, \mathrm{l}$ in the third term, one sees that this term contributes at order $\mathcal{O}(\xi^{3/2})$.

It is easy to verify that the method of Ref. [28] indeed leads to a systematic expansion in ξ by noting that the neglected terms $\Delta f \equiv f - (f_{\mathrm{l}} + f_{\mathrm{h}} - f_{\mathrm{i}})$,

$$\Delta f = -\Phi(\mathbf{p},\mathbf{l}) \, \frac{(b^4 - \mathrm{l}^4/4)\,(b^2 + \mathrm{l}^2/2)}{\mathrm{l}^4\,(\mathrm{l}^2 + \xi\, b^2 + \xi\, \mathrm{l}^2/2)} \, \xi^2 + \Phi_2(\mathbf{p}) \frac{b^6}{(\mathrm{l}^2 + \xi\, b^2)\, \mathrm{l}^2} \, \xi^2 \,, \tag{20}$$

yield contributions to I_1 in Eq. (11) of order of $\mathcal{O}(\xi^3)$ after evaluating the corresponding integrals. At first sight, one might expect that rescaling $\mathrm{l} \to \sqrt{\xi}\, \mathrm{l}$ in the second term effectively lowers the order in ξ, to which this term contributes (naively, to order $\xi^{3/2}$). This is, however, not the case since the leading contribution is canceled by a similar one arising from the first term in Eq. (20). Therefore, Eq. (19) generates all terms in the expansion of the integral I_1 up to and including $\mathcal{O}(\xi^2)$.

For the sake of completeness, we list below terms in the expansion of $f(\mathbf{p},\mathbf{l})$ in powers of ξ, which are responsible for the contributions of orders $\xi^{5/2}$ and ξ^3 to the expansion of I_1:

$$\begin{aligned} f_{\mathrm{l}} + f_{\mathrm{h}} - f_{\mathrm{i}} \;&=\; \dots + \xi^2\, \Phi(\mathbf{p},\mathbf{l}) \left(\frac{1}{8} - \frac{b^6}{\mathrm{l}^6} - \frac{b^4}{2\mathrm{l}^4} + \frac{b^2}{4\mathrm{l}^2} \right) + \xi^2 \Phi_2(\mathbf{p}) \left(\frac{b^6}{\mathrm{l}^4} + \frac{b^4}{2\mathrm{l}^2} \right) + \xi^2 \Phi_4(\mathbf{p}) \frac{b^6}{\mathrm{l}^2} \\ &- \; \frac{\xi^2 \Phi_2(\mathbf{p}) b^4}{2(\mathrm{l}^2 + \xi b^2)} - \frac{\xi^2 \Phi_4(\mathbf{p}) b^6}{\mathrm{l}^2 + \xi b^2} + \frac{\xi^3 \Phi_2(\mathbf{p}) b^6}{2(\mathrm{l}^2 + \xi b^2)^2} \,, \end{aligned} \tag{21}$$

where ellipses refer to terms already given in Eq. (19) and the quantity $\Phi_4(\mathbf{p})$ is defined as

$$\Phi_4(\mathbf{p}) = \frac{1}{4!} \, \frac{1}{5} \, \nabla_{\mathrm{l}}^2 \nabla_{\mathrm{l}}^2 \Phi(\mathbf{p},\mathbf{l}) \big|_{\mathbf{l}=\mathbf{0}} \,. \tag{22}$$

Higher-order contributions in ξ can be systematically obtained along the same lines. In particular, one verifies that the low-l regime yields only terms with half-integer powers of ξ, whereas integer power terms occur from the high-l regime. The intermediate region plays a role of regularization leading to scale-less integrals which cancel the ultraviolet-divergent (infrared-divergent) terms in the low-l (high-l) regimes.

4.2 Results

We are now in the position to apply the expansion method described in the previous subsection. In particular, one immediately observes that there is no recoil correction at order $\xi^{1/2}$. As seen in the previous subsection, non-integer powers of ξ can only appear from the expansion of the integrands in the low-l regime, see Eq. (13). First, we note that the correction ΔI_{st} given by Eq. (10) does not yield the non-integer powers in ξ. Thus, performing the expansion in Eq. (8), the recoil correction for the isovector $\bar{K}N$ case at order $\xi^{1/2}$ reads[4]

$$\Delta I_1 = -I_1 = 2\xi \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\Psi^2(\mathbf{p}) - \Psi^2(\mathbf{p})\right] (\mathrm{p}^2 + \gamma^2) \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{1}{\mathrm{l}^2} \frac{1}{\mathrm{l}^2 + 2\xi(\mathrm{p}^2 + \gamma^2))} = 0 \tag{23}$$

For the isoscalar case, the integral I_{NN} contributes at the same order as I_0. At order $\xi^{1/2}$, the recoil correction ΔI_0 can be rewritten as

$$\Delta I_0 = I_0 + I_{NN} = \frac{1}{M_K} \int \frac{d^3\mathbf{p} d^3\mathbf{q} d^3\mathbf{l}}{(2\pi)^6} \, \Psi(\mathbf{p}) \left[G_{NN}(\mathbf{p}, \mathbf{q}, E(\mathrm{l})) - \frac{\delta(\mathbf{p} - \mathbf{q})}{\mathrm{l}^2/2M_K}\right] \Psi(\mathbf{q}), \tag{24}$$

where G_{NN} is the full NN Green function defined as

$$G_{NN}(\mathbf{p}, \mathbf{q}, E) = \frac{\delta(\mathbf{p} - \mathbf{q})}{\mathrm{p}^2/m_N - E - i0} + \frac{1}{4(2\pi)^3 m_N^2} \frac{M_{NN}(\mathbf{p}, \mathbf{q}, E)}{(\mathrm{p}^2/m_N - E - i0)(\mathrm{q}^2/m_N - E - i0)} \tag{25}$$

and $E(\mathrm{l}) = -\varepsilon_d - \frac{\mathrm{l}^2}{2M_K}$ at this order. On the other hand, the Green function G_{NN} can be expressed in terms of a complete set of the bound- and continuum-state wave functions $\Psi(p)$ and $\Psi_k^{(+)}(p)$, respectively, which are eigenvectors of the two-nucleon Hamiltonian[5]

$$G_{NN}(\mathbf{p}, \mathbf{q}, E) = \frac{\Psi(\mathbf{p})\,\Psi(\mathbf{q})}{-\varepsilon_d - E} + \int d^3\mathbf{k} \frac{\Psi_k^{(+)}(\mathbf{p})\Psi_k^{(+)\dagger}(\mathbf{q})}{\mathrm{k}^2/m_N - E - i0}. \tag{26}$$

The wave functions $\psi(\mathbf{p}) = \{\Psi(\mathbf{p}); \Psi_k^{(+)}(\mathbf{p})\}$ satisfy the Schrödinger equation

$$\left(\epsilon - \frac{\mathrm{p}^2}{m_N}\right) \psi(\mathbf{p}) = -\frac{1}{4m_N^2} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} V(\mathbf{p}, \mathbf{p}')\psi(\mathbf{p}'), \tag{27}$$

with $\epsilon = \{-\varepsilon_d; k^2/m_N\}$, respectively. The normalization for the potential is chosen so that the Lippmann-Schwinger equation for M_{NN} is given by

$$M_{NN}(\mathbf{p}, \mathbf{q}, E) = V(\mathbf{p}, \mathbf{q}) - \frac{1}{4m_N^2} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \frac{V(\mathbf{p}, \mathbf{p}')M_{NN}(\mathbf{p}', \mathbf{q}, E)}{E - \mathrm{p}'^2/m_N + i0}. \tag{28}$$

Substituting Eq. (26) into Eq. (24) one obtains

$$\Delta I_0 = 2 \int \frac{d^3\mathbf{l} d^3\mathbf{k}}{(2\pi)^6} \, \frac{1}{\mathrm{l}^2 + 2\xi(\mathrm{k}^2 + \gamma^2)} \left| \int d^3\mathbf{p} \; \Psi(\mathbf{p}) \, \Psi_k^{(+)\dagger}(\mathbf{p}) \right|^2 = 0, \tag{29}$$

[4]The integral over l yields a contribution $\sim 1/\sqrt{\xi}$ leading to the correction for $I_{0,1}$ at order $\xi^{1/2}$.

[5]Note that the high-l regime is perturbative, whereas the low-l regime is not. Under this we mean that, e.g., in the low-l regime one has to consider the full NN amplitude in Eq. (25) without expanding it in Born series – since all terms in this expansion contribute at the same order in ξ. On the other hand, if we are extracting integer powers of ξ in the high-l regime, the perturbative treatment of the nucleon-nucleon scattering amplitude is legitimate, see Eq. (30).

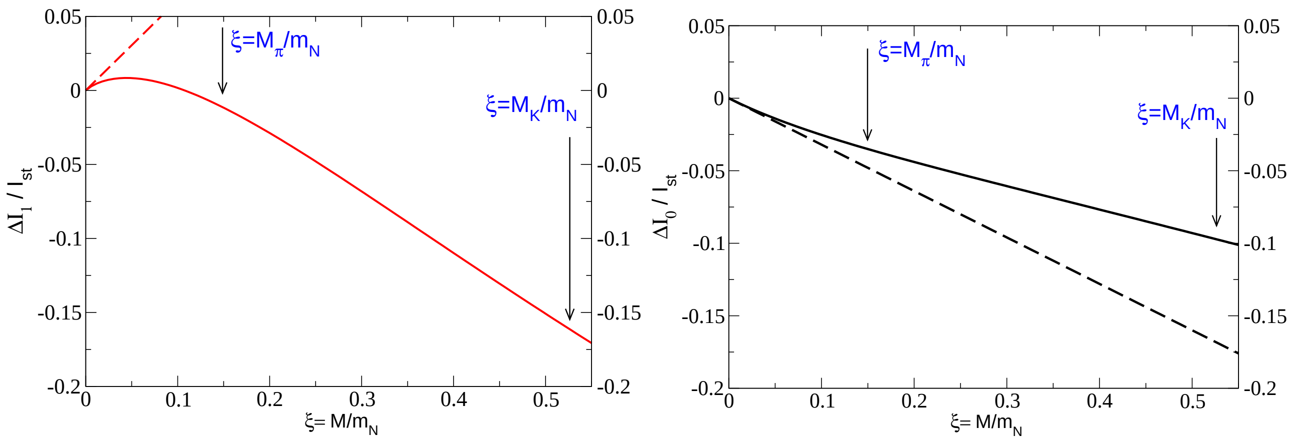

Figure 2: Recoil corrections in the double-scattering process for the isovector (left panel) and isoscalar (right panel) cases. The solid curves correspond to the results of full numerical calculations (see text for more details), the dashed curves represent the first non-vanishing recoil corrections at order ξ. The arrows indicate the results for πd- and $\bar{K}d$-scattering.

where we exploited the othogonality of the bound- and continuum-state wave functions and made use of the normalization of $\Psi(\mathbf{p})$ according to $\int d^3\mathbf{p}\,\Psi^2(\mathbf{p}) = (2\pi)^3$. Thus, at order $\xi^{1/2}$ there is a complete cancellation of the recoil corrections both for isoscalar and isovector types of $\bar{K}N$ interaction. The origin of the cancellation for the isovector case is explained by the Pauli principle (see Ref. [26]), whereas for the isoscalar case it is the orthogonality of the bound-state (deuteron) and continuum-state (NN intermediate state) wave functions in the $^3S_1$ partial wave. Note that for πd-scattering similar cancellations were observed in Ref. [25] using a potential-model approach.

The non-vanishing recoil corrections to the static term appear at order ξ both for isoscalar and isovector $\bar{K}N$ interactions. In order to perform an analytic study of this and higher-order terms in the expansion we choose NN interaction in the separable form $V(\mathbf{p},\mathbf{p}') = \lambda g(\mathrm{p})g(\mathrm{p}')$, where $g(\mathrm{p}) = (\mathrm{p}^2+\beta^2)^{-1}$, $\Psi(\mathbf{p}) = Ng(\mathrm{p})(\mathrm{p}^2+\gamma^2)^{-1}$, $N = \sqrt{8\pi\gamma\beta(\gamma+\beta)^3}$, and $\beta = 1.4\ \mathrm{fm}^{-1}$.

Performing the Fourier transform and making use of the Schrödinger equation, one obtains the following result for the linear corrections:

$$\begin{aligned}\Delta I_1^{\xi} &= \frac{\xi}{4\pi}\int d^3\mathbf{r}\, r\Psi(\mathbf{r})(\gamma^2-\Delta)\Psi(\mathbf{r}),\\ \Delta I_0^{\xi} &= \frac{\xi}{4\pi}\int d^3\mathbf{r}\, r\Psi(\mathbf{r})(\gamma^2-\Delta)\Psi(\mathbf{r}) - \frac{\xi}{16\pi m_N}\int d^3\mathbf{r}d^3\mathbf{r}'\,\Psi(\mathbf{r})\Psi(\mathbf{r}')V(\mathbf{r},\mathbf{r}')|\mathbf{r}-\mathbf{r}'|\,. \end{aligned} \tag{30}$$

Evaluating these terms for the employed NN interaction we get

$$\frac{\Delta I_1^{\xi}}{I_{st}} \approx 0.6\,\xi \quad \text{and} \quad \frac{\Delta I_0^{\xi}}{I_{st}} \approx -0.3\,\xi\,. \tag{31}$$

In Fig. 2 we show the results for the recoil correction (in units of the static term I_{st}) for isovector (left panel) and isoscalar (right panel) contributions as a function of ξ. The results of our full numerical calculation of the recoil corrections, see Eqs. (8), (9) and (10), without expanding in ξ are shown by the solid curves. Surprisingly, even for $\bar{K}d$ scattering the nucleon recoil effect turns out to be not that large as one could *a priori* guess. As can be seen from the figure, the nucleon recoil for the double-scattering process amounts just to 10-15% of the static contribution. To understand the origin of the

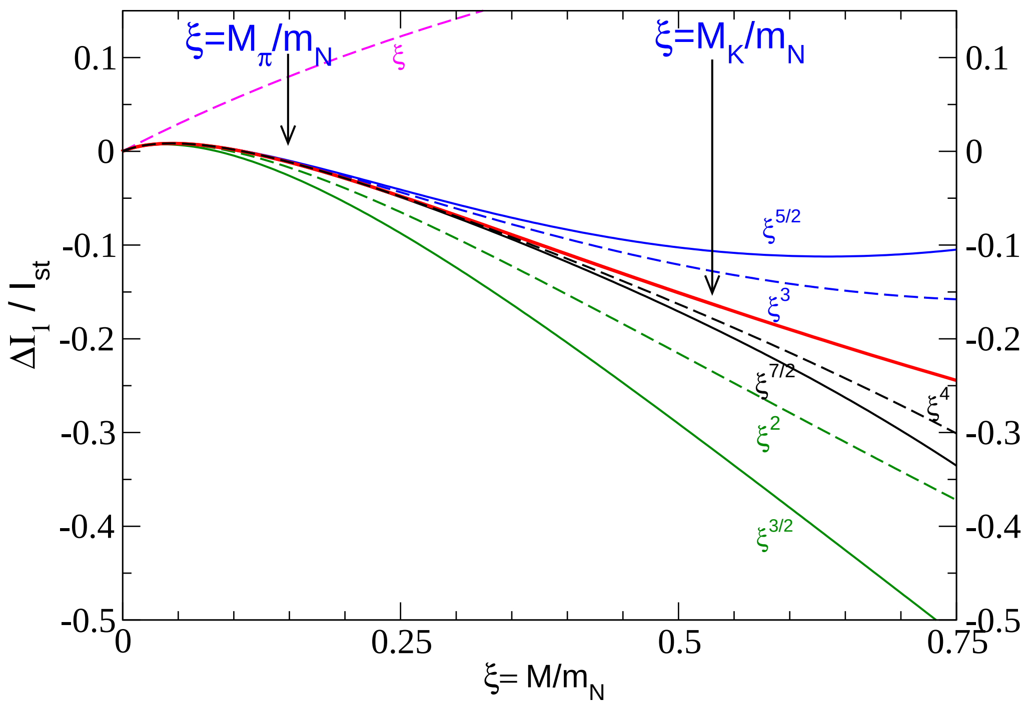

Figure 3: Convergence of the expansion in ξ for the recoil corrections in the double-scattering process for the isovector $\bar{K}N$ interaction. The notation is explained in the text. The arrows indicate the results for πd- and $\bar{K}d$-scattering. Note that the (trivial) kinematical pre-factor $1/(1+\xi)$ was not expanded in ξ (see the text for details).

smallness of the effect we compare the full result with the EFT calculation based on the expansion in powers of ξ. Our results at order ξ are shown by the dashed lines. As visualized in Fig. 2, the full recoil correction changes its sign in the considered interval of ξ for the isovector case. Here, the linear approximation fails completely to describe the full result. On the other hand, for the isoscalar case the recoil correction has a constant sign, and the linear approximation yields a reasonable result.

To further explore the convergence of the expansion in ξ in the case of the isovector $\bar{K}N$ interaction we calculated higher-order corrections using the expansion method described above. The results are shown in Fig. 3. The thick red line corresponds to the full result, solid lines represent the calculations up to and including half-integer powers of ξ whereas the results shown by dashed lines include, in addition, the next-higher integer power of ξ. Fig. 3 demonstrates that already at order ξ^2 one reproduces the bulk of the effect while the order-ξ^4 calculation already provides a very good approximation to the underlying result for $\bar{K}d$ scattering. One can see from this figure that the smallness of the net recoil effect is accounted for by the specific cancellation pattern among different recoil corrections. In particular, there is a huge cancellation between the first integer (at order ξ) and the first non-integer (at $\xi^{3/2}$) corrections that even leads to the change of sign for the recoil effect. Further, while improving convergence at smaller ξ, the inclusion of higher-order half-integer terms results in an oscillatory behavior around the full result at larger ξ. Actually, looking at Eqs. (19) and (21), one can already see that the sign in front of the leading non-integer terms changes while going from order $\xi^{3/2}$ to $\xi^{5/2}$ (cf. the last term in Eq. (19) and the last three terms in Eq. (21)). This can be explained by considering the expansion of the 3-body propagator in Eq. (11) in the low-momentum

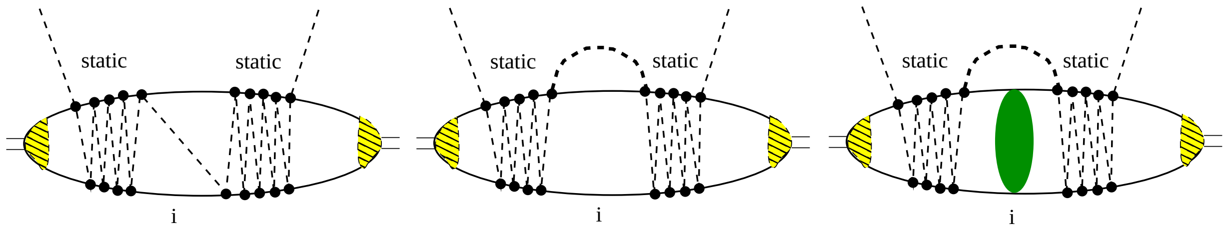

Figure 4: Inclusion of the recoil corrections in the multiple-scattering process.

regime, which leads to an alternating series

$$f = (\mathrm{l}^2\Phi_2 + \mathrm{l}^4\Phi_4 + \ldots)\,\frac{\mathrm{l}^2/2 - b^2}{\mathrm{l}^2 + \xi b^2}\left(1 - \frac{\xi \mathrm{l}^2}{2(\mathrm{l}^2 + \xi b^2)} + \frac{\xi^2 \mathrm{l}^4}{4(\mathrm{l}^2 + \xi b^2)^2} - \ldots\right). \tag{32}$$

Due to this pattern, terms with half-integer (HI) powers of ξ contributing to I_1 also show alternating behavior

$$I_1^{\mathrm{HI}} = \frac{1}{1+\xi}(2.1\xi^{3/2} - 0.96\xi^{5/2} + 0.85\xi^{7/2} - 0.81\xi^{9/2} + 0.8\xi^{11/2} - 0.8\xi^{13/2} + \ldots)\,, \tag{33}$$

which results in their partial cancellation. Notice further that the (trivial) pre-factor $1/(1+\xi)$ in I_1, see Eqs. (11) and (33), produces large coefficients when expanded in ξ. After expanding it in powers of ξ, Eq. (33) turns into

$$I_1^{\mathrm{HI}} = (2.1\xi^{3/2} - 3.07\xi^{5/2} + 3.91\xi^{7/2} - 4.72\xi^{9/2} + 5.53\xi^{11/2} - 6.33\xi^{13/2} + \ldots)\,. \tag{34}$$

The convergence in the latter case is much slower (albeit the series still converges at the value of ξ corresponding to the kaon mass). For this reason, we kept this trivial kinematical pre-factor unexpanded when showing our results in Fig. 3.

Finally, we would like to emphasize that due to large cancellations between individually sizable terms discussed in this paragraph, the recoil effect turns out to be not that large even for $\bar{K}d$ scattering. We found that it is about 10-15% of the static piece for the separable model of NN interaction (see arrows that indicate the results for $\bar{K}d$ scattering in Fig. 2).

5 Recoil effect in multiple scattering

Due to the strong $\bar{K}N$ interaction, the recoil effect in the multiple-scattering diagrams might be significant and must be studied quantitatively. Moreover, it can be shown that, starting from the quadruple $\bar{K}N$ scattering process, the recoil correction becomes nonzero already at order $\sqrt{\xi}$. A detailed discussion of the recoil effect in the multiple scattering will be reported elsewhere [30]. Here we just sketch the method. The crucial assumption of the method is that the recoil corrections can be treated perturbatively, even if the static $\bar{K}N$ interactions have to be resummed to all orders. Thus, one may study an insertion of 1,2,... "retarded" blocks in the diagrams that contain infinitely many static kaon propagators. An example is shown in Fig. 4, where we have dressed the double-scattering block – studied in detail in the previous section – by static kaon rescattering in the initial and final states. In order to keep track of various powers of ξ within this method, it is essential to have an explicit perturbative expansion of the retarded block that can be achieved by using the technique described in the present article.

6 Conclusions

We have studied the nucleon recoil effect for $\bar{K}d$ scattering using EFT. Specifically, using the expansion method of the Feynman diagrams in EFT, we have calculated recoil corrections to the double-scattering process in a systematic expansion in the half-integer powers of the parameter $\xi = M_K/m_N$. It was shown that the leading correction to the static term, which emerges at order $\xi^{1/2}$, cancels completely both for isoscalar and isovector types of $\bar{K}N$ interaction. The origin of the cancellation for the isovector case can be explained by the Pauli principle whereas for the isoscalar case it stems from the orthogonality of the bound state (deuteron) and continuum (NN intermediate state) wave functions in the $^3S_1$ partial wave. The coefficients of higher order terms in the expansion in $\xi^{1/2}$ appear to be of a natural size and the series converges even for the value of ξ corresponding to the physical kaon mass. Due to a significant cancellation that takes place between individually sizable terms at different orders, the net recoil effect for the double-scattering process is found to be just about 10-15% of the static contribution for the separable model of NN interaction. We also sketched the method that can be used to include the nucleon recoil for the multiple-scattering process. A more detailed discussion of this and other aspects will be presented in the forthcoming publication [30].

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